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Research Article

Numerical Study to Assess the Structural Behavior of the Bajrakli Mosque (Western Kosovo)

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Cultural heritage is one of most significant concerns in modern societies carrying different social and historical values. Among the stock of cultural heritage, historical monuments are one of the greatest contributors to the values in many aspects. Due to several factors, such structures have gone through changes causing structural deficiencies. The aim of this study is to provide a clear insight of the cause and impacts of structural deficiencies through visual inspections and computational methods. As a representative model, Bajrakli Mosque located in western of Kosovo is selected as a case study. During visual inspections, some cracks are found along the structural elements of the mosque. A possible cause of the structural cracks may be the stress concentration through the regions of the structure. In order to provide a better understanding, two different loadings are considered to examine the structural behavior of the mosque. The first loading covers the analysis due to gravity loads, whereas the second one defines the dynamic loading due to ground shakings defined by the earthquake spectrum using finite element analysis in SAP2000. By means of these analyses, the performance of the building is examined. As a result, important data are obtained for identifying the critical regions of the structure. The maximum displacement of the structure is found to be 7.1 mm and 8.0 mm in combination of self-weight and earthquake load in X and Y direction, respectively. Moreover, the regions showing highest values of stress concentration are found through the small domes, through the openings of main dome and connections with arches, and around the openings of the walls.

1. Introduction

Determining the seismic response of existing structures is a very important issue for the evaluation of their seismic vulnerability and is gaining importance for historical monumental buildings, especially if they are characterized by complex geometrical layouts, such as in the case of historical mosques. Reconnaissance studies carried out after earthquakes have proven how sensitive mosques are to horizontal earthquake forces [1–7].

Monumental buildings such as mosques, churches, castles, city walls, and clock towers constructed in many parts of the world are the main motifs of historical masonry structures [1, 8]. They represent an important part of Balkan cultural heritage, particularly prone to damage and susceptible to partial or total collapse under earthquake loads as testified by some of the recent earthquakes (Macedonia Earthquake in 1963; Banja Luka, Bosnia, and Herzegovina in 1969; Friuli, Italy in 1976; Coast of Montenegro in 1979; Durres, Albania in 2019) in the region [4, 9–11].

These structures are one of the most important components of our cultural diversity reflecting the history of humanity. Thus, preservation and restoration of these structures is an important engineering concern and task for us to ensure the sustainable development and protection of our cultural resources to pass them onto future generations [12, 13].

Specific material characteristics (highly inelastic response and small tensile strength), lack of proper connections between various elements, presence of thrusting elements, and particular configuration of these buildings that are characterized by open plan layouts with slender structural components make these structures seismically vulnerable [14–17].

Analysis approaches developed for modern framed structures may not be feasible for historical monumental buildings [18]. Adequate experience of old construction technical concepts, understanding of structural response, good engineering judgment, and accurate interpretation of comprehensive structural analyses results are required for appropriate intervention [19–21]. While masonry structures can generally carry vertical loads very safely and stably, they are structurally very sensitive to horizontal loads [22]. Each monumental building is a unique one, characterized by its own history. The dynamic behavior of historic buildings is too complex to be interpreted by simple mechanical models [23]. Reliable quantitative strength assessments are difficult to make due to the difficulty of collecting experimental data on the strength of structural components and even the mechanical characteristics of materials on-site [24].

Limitations on the ability to inspect the building or difficulties arising in extracting specimens from buildings of historic value (as well as the associated high costs for diagnosis) often result in limited knowledge of the internal structure system or the properties of existing materials. Also, degradation and deterioration of the material resistance during the lifespan of ancient structures is frequently encountered by avoidance.

The inherent literature covers a plethora of illustrative case studies that reports a wide range of applications on such structures [4, 5, 10, 25–35]. Balkans has been the cradle of many civilizations throughout the history because of its location, acting as a bridge between two continents. Each civilization had left its physical signs in the form of small objects to large-scale buildings and sometimes even to almost complete cities. Some of the remains of the past have been able to survive until our time and the others have been lost in the course of time partly by human activities and other natural disasters [10, 36, 37].

Due to the importance of historical monuments, the present study discusses the structural behavior of the historical stone masonry Bajrakli Mosque in Peja/Kosovo, having the main elements as a representative of the Ottoman Mosques, by following up qualitative (such as site inspection and historical research) and quantitative (such as structural analysis and monitoring) approaches. Such structures emphasize the role and significance of civil engineering in other areas of life. These monuments carry important information to historical events, characters, and historical development. They play a huge role in defining engineering and architectural eras through the history, including the advance of the construction techniques, ornamental models, construction materials, and many other related issues. First, a detailed visual inspection is done on the site to detect the cracks and possible deficiencies after a historical study on the construction of the mosque. Second, a global analysis of the mosque has been made by using the finite element (FE) modeling technique. Then, the obtained results are compared with visual inspection observations on load-bearing elements of the mosque. It is believed that the results reported here could provide a representative case study that extends the understanding of the structural response of this building typology. Considering its value in historical phases, construction advances, social and religious importance, and the scarcity make this study as a significant and interesting topic.

2. Historical Bajrakli Mosque

The historical buildings in Kosovo and Albania are documents of the importance of Albanian population in Ottoman periods. The Bajrakli Mosque was built in the center of Peja, western Kosovo, at the so-called Old Bazaar, an outdoor market which is a characteristic of Albanian cities (Figure 1 [38]). The mosque was a legacy of Sultan Mehmet Fatih who also supported the building of Grand Mosque in Prishtina. The Bajrakli Mosque was built on 1471. It is a monumental old building consisting of all elements. The mosque is made of the prayer hall, entrance hall, and a minaret. The whole structure is covered by a big dome, covering the prayer hall. Also, the entrance hall is covered by three small domes of commonly proportional characteristics. The interior of the mosque is covered by wall paintings and citations from Quran. It also consists of other elements such as sculptures, framing of portal, and windows. In the serving history of the mosque, it is important to mention that the mosque was burnt by Serb forces during the war of 1998-1999. The restoration of mosque was done after the war, at both architectural elements and the sculpture paintings [Trashigimia Kulturore *e* Kosoves, 2016].

The Mosque has a rectangular plan with a length of 18.87 m and width of 13.95 m (Figures 1 and 2 [39]). From the total area of 263.24 m^2, 68.77 m^2 belongs to the entrance hall, while the rest belong to the prayer hall. The prayer hall is covered by the main single dome, while the entrance hall is covered by three small domes. The highest part of the mosque is the minaret, with a height of 28 m, while the dome reaches about 13 m. In the north façade, there are four marble columns at which the small domes are supported on. Figure 2 shows the geometrical shapes and elements composing the mosque.

3. Damage Survey

On-site inspection plays an important role in the seismic assessment of historic structures, which aims to determine the building's condition and define a representative structural model. At the time of the inspections, an architectural restoration had been completed.

Stone is one of the oldest materials used in construction, and for this reason, many historical monuments are built of stone, including the structure selected for this study. Considering the age of the structures, stone has shown a good performance as a construction material. For the time being built, the structures were constructed without any rule or standardized regulations, without any research or study, relying only in the experiences from the past. Due to seismic activities, temperature changes, weather conditions, and inappropriate maintenance, those structures have gone through different changes.

Figure 1: Bajrakli Mosque [38] and its location in Kosovo (Encyclopedia Britannica, Inc.).

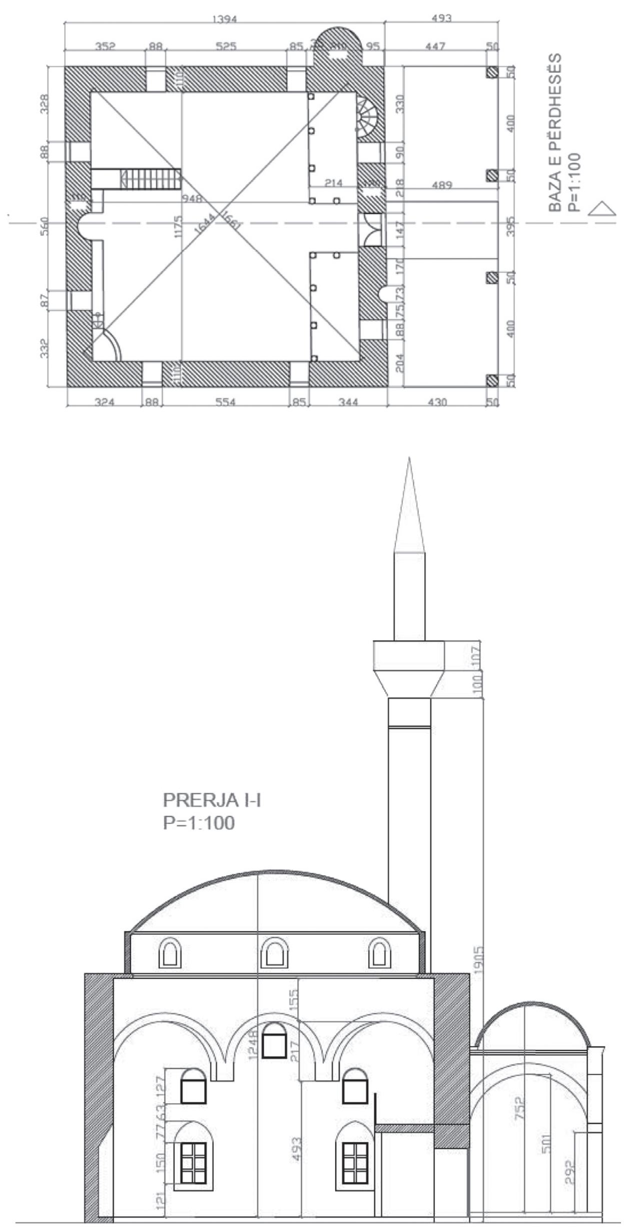


Figure 2: Geometry of the Mosque: plan and north elevation [39].

The first examination of the structure is carried out by visual inspections. The inspection consisting of a geometrical relief also aimed at a quality check-up of element connections and characterization of the masonry texture. This stage was accompanied by a historical survey of the mosque to verify the original geometry and to consider whether the mosque underwent modifications over time. Then, the mathematical model has been developed to reproduce the geometry of the mosque, concentrating on the details of the connections and openings.

It is important to check for deficiencies that can be determined visually. As the geometrical data of Bajrakli Mosque are obtained, the structure is visited at site and cracks, and other irregularities are identified. Because of architectural restorations, structural deficiencies cannot be identified easily. However, during the site inspections, some of the cracks could be seen.

While checking in the interior of the structure, there are some cracks in the domes, pendentives, and arches. Considering the fact that those cracks can not only be plaster cracks but structural cracks as well, it is important to identify them as structural deficiencies. Cracks may have been developed from different factors such as lack of maintenance, aging of the materials, weather conditions, improper isolation, moisture, and earthquake loads. Figure 3 shows the interior cracks of domes, arches, and pendentives.

Generally, damages in masonry structures occur in the main body of load-bearing systems. From site inspections, cracks can be identified in the load-bearing walls of the structure. Many of the cracks can be seen close to the edges of the windows and extending along the walls. Some of the reasons for the developed cracks in load-bearing walls are such as the compressive stress as a result of vertical loads, shear stress as a result of lateral force, tension forces, and cracks due to earthquake forces. Figure 4 shows the cracks on the bearing walls of the structure. Given the current crack patterns, the stability of the main dome is uncertain for possible future earthquake shakings.

During the site inspections, it is seen that stone masonry blocks used for building the Bajrakli Mosque show decay and degradation to the natural buildings materials as shown in Figure 5.

On the other hand, porosity of materials is an important factor affecting the stony masonry that may lead into frost attack and presence of soluble salt in masonry [40–45]. Frost attack is one of the major agents that cause the eroding of masonry structures. It is a common problem that is caused with the movement of water to the pores. During the low temperatures, the water turns into ice and thus results into an increase in its volume, causing extra stress within masonry [46].

Salt content is one of the main factors causing the decay of porous material, which is a major concern for the preservation of historic buildings. Salt is an element which can be transported by water and can be accumulated in the pores. Salt's behavior is unpredictable since they can remain inactive for some time, and later, they can become active and cause weakening surfaces and natural decaying.

4. Analytical Modeling of the Structure

4.1. Generation of the Finite Element Model. As briefly discussed in introduction part, determination of the seismic response of historical masonry buildings is a challenging task due to understandable reasons comprising the incomplete characterization of the mechanical properties of the material, difficulties in numerical modeling, and the complex architectural layouts [47]. In the scientific literature, some mechanical models were proposed to correctly estimate the response of masonry material that adopt the various strategies. This study considers the numerical problem using FEM modeling approach and to assess the seismic response of the mosque.

Mathematical modeling is a significant phase in the analysis of historical masonry structures. The three-dimensional FEM model developed by the SAP2000 software package [48] has been deployed by using a set of finite elements. The structure may have gone through changes that generally are not documented. Thus, sometimes, it may be difficult to identify structural elements from ornamental ones. Considering the availability of the sources, a method of modeling should be selected for defining the state of preservation for the structure to be restored. Figure 6 [49] shows the three modeling approaches for the modeling of masonry structures.

Micromodeling: units and mortar in the joints are considered by continuum components, whereas the unit–mortar interface is characterized by discontinuum elements in this approach (Figure 6(a)). While this strategy leads to more precise results, the level of sophistication and the corresponding analysis is computationally demanding, limiting its use to small-scale laboratory specimens and structures.

Mesomodeling: in this approach, the masonry units are simulated as fictitious extended parts by continuum elements with the same size as the original bricks' dimensions combined with the actual joint thickness. The mortar joint is also modeled as an interface with zero thickness as shown in Figure 6(b). According to this technique, the properties of the mortar and the mortar/unit boundary are concentrated into a common element, while expanded elements are used to characterize the brick units. This method leads to the reduction of the computational cost and yields a model that is applicable to a wider range of structures.

Macromodeling: in this approach, bricks, mortar, and the brick–mortar interface are smeared out in a homogeneous continuum (Figure 6(c)). Masonry is considered as a homogeneous, isotropic, or anisotropic continuum material which can be represented by phenomenological models. The impact of existing mortar joints as the main source of weakness and inelasticity cannot be addressed using this strategy. While this method may be ideal for the analysis of large-scale masonry structures, it is not appropriate for the detailed stress analysis of small masonry panels due to the difficulty of capturing all its anticipated failure modes.

The selected modeling technique is macromodeling, which is the most used strategy for studying large-scale structures and the effect of global factors [50].

Figure 3: Crack inspection at domes, arches, and pendentives.

Figure 4: Crack inspection at load-bearing walls.

Figure 5: Degradation of stone on masonry piers.

Structures such as historical monuments are usually more complex and challenging due to different elements composing the structure. Each part of the structure has its own techniques and its own difficulties, and each of them affects the overall behavior of the building. As stated previously, the Bajrakli Mosque is composed of a main dome,

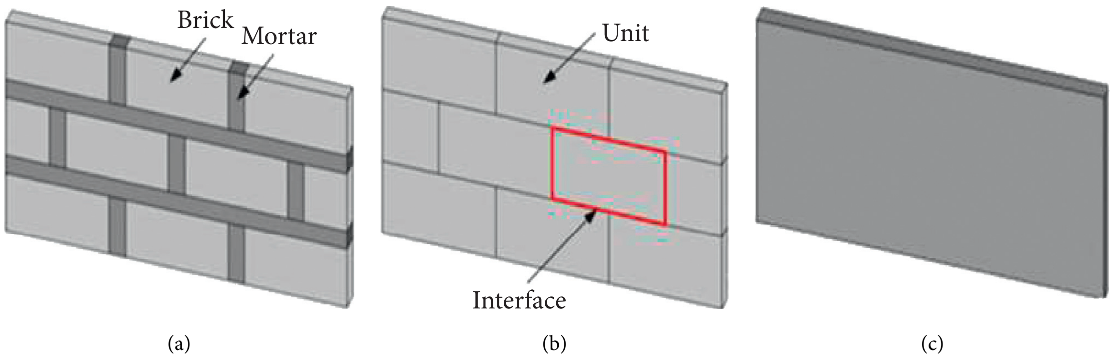


Figure 6: Masonry modeling approaches. (a) Micromodeling. (b) Mesomodeling. (c) Macromodeling [47, 49].

four semidomes, three small domes, arches, pendentives, walls, and columns. Modeling of such monuments with domes such as mosques should start with modeling of the roof system first. Figure 7 [25] shows some finite element models of roof systems of Ottoman architecture from the study "Structural Analysis of Domed Roof Systems in Architect Sinan's Works" by Bilgin [25].

As the main dome is modeled, arches, semidomes, and pendentives are defined. Semidomes are more challenging than main dome, since it requires understanding the 3D system of the modeling provided by SAP2000. The following figure shows the mentioned key data that should be defined for semidome modeling. Figure 8

The first step is to locate the origin according to three-dimensional modeling. The origin coordinates should be precise; otherwise, it can cause error in load transferring and wrong output data after analysis. Another important step is the number of Z division and number of angular divisions. Those should match with the meshing system created from the main dome and arches. After the roof system is modeled, walls and other elements can be defined. Hinge supports are used for the base of the structure.

After a preliminary in situ survey of the mosque to measure basic information on geometry, the structural detailing, and for any possible irregularities, a representative structural scheme of Bajrakli Mosque was simulated through a 3D finite element numerical model using the SAP2000 software package.

A simplified geometry of the structure was adopted in the analytical modeling, following the macromodeling approach as adopted in various studies [30, 51–53], since a 3D numerical model with fine meshing is preferable to simulate the response of monumental structures with complex architectural geometry [54]. The definition of the geometry was achieved through the provided architectural layouts and site measurements. The finite element model of Bajrakli Mosque shown in Figure 9 consists of 9690 nodes; 9515 shell elements for the main dome, arches, semidomes, and small domes; and 4 frame elements for the columns in the entrance hall. The mathematical model includes main load-bearing volume as shown in Figure 9.

5. Analysis Parameters

5.1. Material Properties. Considering the values of historical structures, analyzing may be difficult. Some of the points which make the analysis of historical buildings a complex task can be classified as follows [47]:

(i) Geometry records may be rare or cannot be found at all

(ii) The data about the inner core of the elements found in the structure may be lost

(iii) Classification and description of the material properties used is challenging and costly

(iv) Huge inconsistency of mechanical properties due to work quality and use of natural resources

(v) Substantial changes in the core of and composition of essential elements of the structure due to long periods of construction

(vi) The damages caused in the structure may not be totally detected

The materials are assumed to be isotropic and homogeneous considering the typology of the masonry. Due to the technical limitations, it was impossible to conduct tests on the main characteristics of the mosque materials. Thus, following input parameters (Table 1) were selected according to the indications provided in previous research and suggestions for existing mosques and cultural heritage. Therefore, similar experimental and analytical studies in the relevant literature have been studied in detail and given in Table 2.

The adopted material characteristics are done based on the materials that may represent the structure according to the time being built and its similarity for this type of structure. Table 1 provides the selected material properties for this study.

The other three characteristics in this study are the allowable compressive, allowable tensile strength, and allowable shear stress. According to those characteristics, the mosque structure is analyzed and checked for drawing final conclusions.

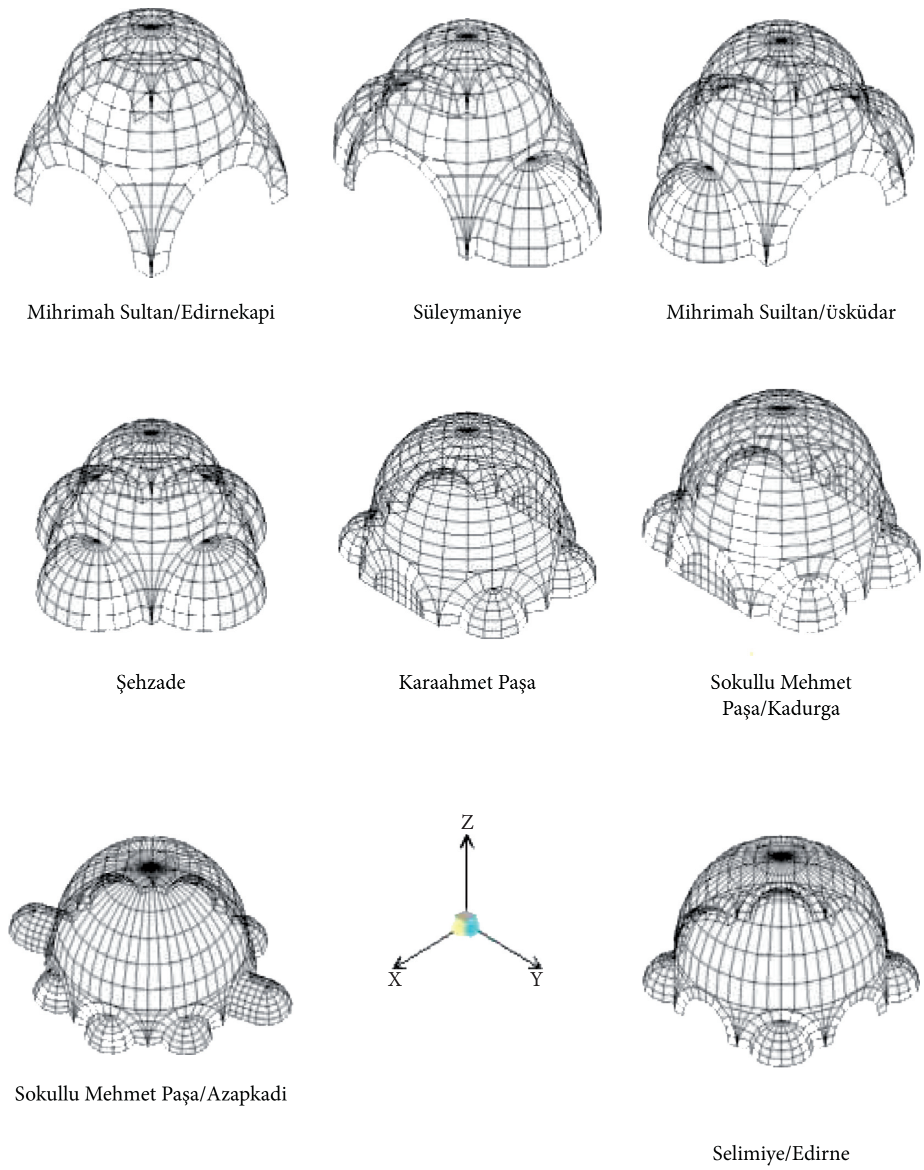


Figure 7: Finite element models of domed roof systems [25].

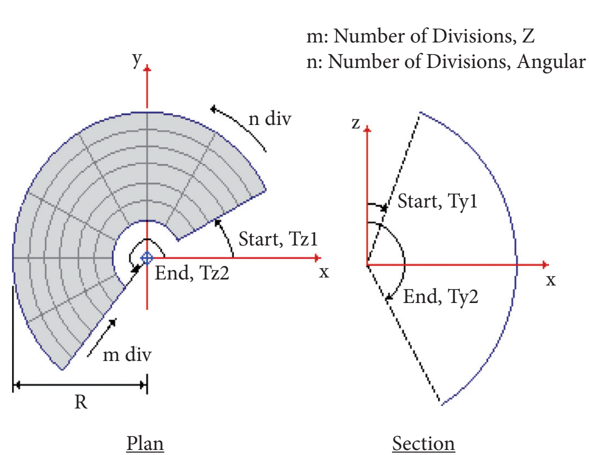


Figure 8: Inputs of modeling semidomes.

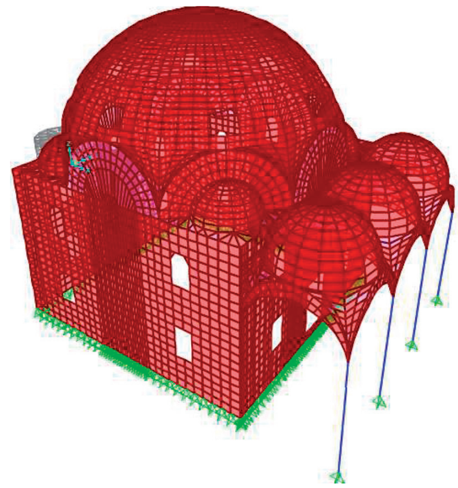


Figure 9: 3D FEM model of the Bajrakli Mosque.

Table 1: Selected material properties.

Characteristics	Stone
Unit weight γ (kN/m^3)	21 [9].
Modulus of elasticity E (MPa)	450 [19].

It is also important to note that during earthquake load analysis, no reduction was made to earthquake load, and reduction factor is considered as $R = 1$. As the authors Beeson et al. [19] stated in their study "Seismic Vulnerability of Structures with Irregular Symmetry," Turkish Earthquake Codes suggest the compressive allowable stress as $\sigma_{\text{all}} = 0.3$ MPa. The authors suggested that those values to be tripled and used as the limits. The following equations are proposed for determining the allowable compressive, tensile stresses, and shear stresses [19]:

$$\begin{aligned}\sigma_{m(\text{compressive})} &= \sigma_{\text{all}}x3,\\ &= 0.3x3 = 0.9\text{MPa}.\end{aligned} \tag{1}$$

While, the tensile strength is assumed to be as 15% of the specified allowable compressive stress, and it is defined as the following equation [19]:

$$\begin{aligned}\sigma_{m(\text{tensile})} &= 0.15,\\ &= 0.9x0.15 = 0.135\text{MPa}.\end{aligned} \tag{2}$$

The limit values for shear stress are defined from the following equation, in which τm is the wall limit stress, t_o is the allowable wall failure stress and assumed to be 0.3 MPa, μ is the friction coefficient (assumed to be 0.5), and s is the wall vertical stress, which are given in Table 3 [19].

$$\tau_m = \tau_o + \mu \cdot \sigma, \tag{3}$$

where $\sigma = \sigma_{m(\text{compressive})}/2$

The limit shear stress value for stone in domes, arches, and walls resulted to be

$$\tau_m = 0.3 + 0.5x\left(\frac{0.9}{2}\right) = 0.53\text{MPa}. \tag{4}$$

The dynamic analysis of Bajrakli Mosque involves the response spectrum which is selected based on Eurocode 8 with 0.25_g acceleration, as shown in Figure 10.

6. Modal Eigenvector Analysis

The three-dimensional numerical model has been used to evaluate the modal response of the Bajrakli Mosque. Thus, the analysis provides the data for deformational behavior and stress distribution throughout the structure. The first type of analysis is the eigenvector modal analysis. It shows free vibration mode shapes and through periods and frequencies of the system.

Figure 11 shows the first five modal shapes derived from modal eigenvector analysis. The 1$^{\text{st}}$ mode shape of the mosque involves the translation along the weakest transversal direction, with significant out-of-plane deformation of the orthogonal components. Table 4 presents the data for the first five modal shapes of the mosque.

7. Static and Dynamic Analyses

This section discusses about the analysis for shell stresses under static and dynamic loads. In this study, linear analysis is used. Nonlinear analysis in historical masonry structures is complex and considered as "meaningless" in practical engineering applications for such types of structures [59].

With the linear analysis, deformational behavior and stress distribution of the structure are found. This type of analysis has been widely used to analyze historical masonry structures. Linear elastic analysis assumes that the material obeys Hook's law [60].

Static loads are represented by the dead load or self-weight of the structure, while dynamic loads are represented by earthquake loads. All of the results are highly dependent on the macromodeling phase of the structure.

For the results to be more accurate, two different loading conditions are considered for the linear analysis of Bajrakli Mosque, $G + \text{EQ}_x$, and $G + \text{EQ}_y$ for gravity load and earthquake load in x and y direction, respectively. These loading

Table 2: Material properties from previous studies.

Properties	Mustafaraj [9]	Gedik and Celep [55]	Hrasanica and Medic [56]	Ustundag et al. [57]	Beeson, Kubin, Unay [19]	Sepetçi [58]
Unit weight γ (kN/m^3)	21	20	20	17.658	24	24
Modulus of elasticity E (MPa)	1740	2000	3000–5000	2000	450	450
Void ratio ν	0.2	0.2	—	—	—	0.2
Tensile strength (MPa)	1.42	—	—	—	0.135	—
Compressive strength (MPa)	4.06	—	5	—	0.9	—

Table 3: Selected allowable stresses.

Characteristics	Allowable compressive stress (MPa)	Allowable tensile stress (MPa)	Allowable shear stress (MPa)
Stone	0.9	0.135	0.53

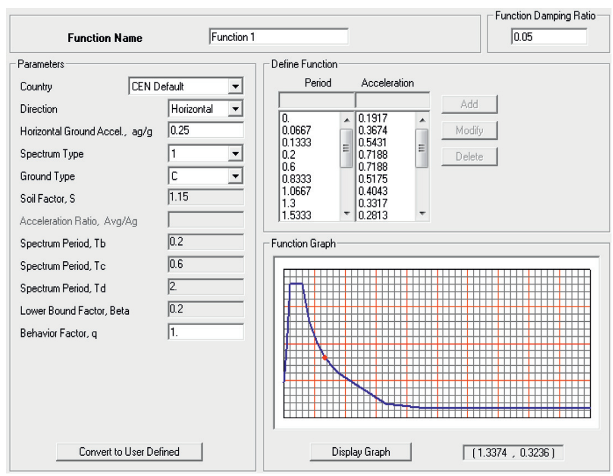
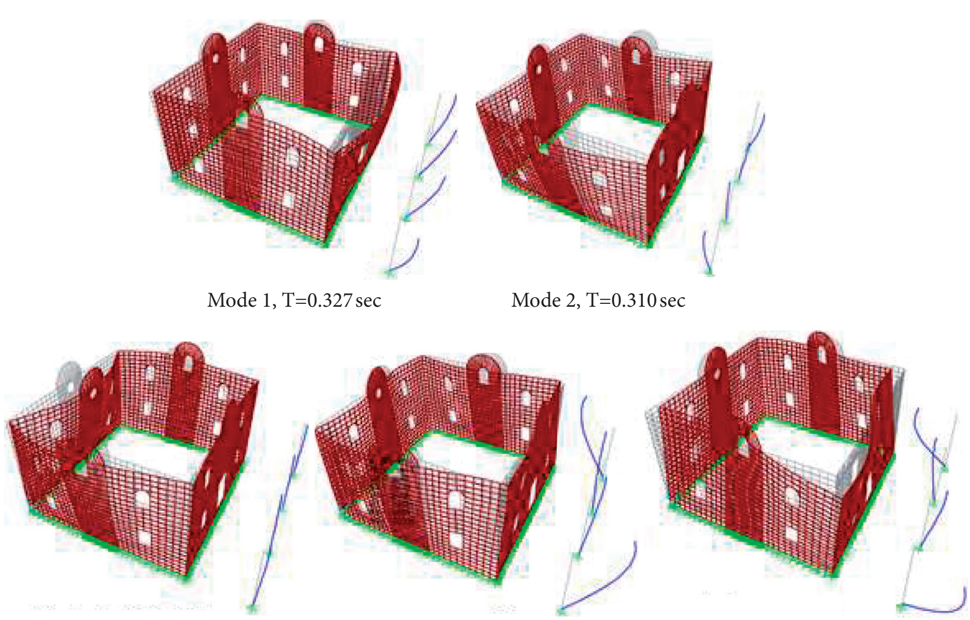


Figure 10: Response spectrum inputs.



Mode 1, T=0.327 sec

Mode 2, T=0.310 sec

Mode 3, T=0.301 sec

Mode 4, T=0.234 sec

Mode 5, T=0.197

Figure 11: Modal shapes of the structure.

Table 4: Modal data of the structure.

Mode	Period (sec)	Mass participation x	Mass participation y	Mass participation z
1	0.327	0.7467	0.0003	0.0006
2	0.310	0.0011	0.6446	0.0000
3	0.301	0.2094	0.0012	0.0000
4	0.234	0.0000	0.0048	0.0000
5	0.197	0.0000	0.0071	0.0000
6	0.182	0.00709	0.0000	0.0177
7	0.170	0.02653	0.0000	0.0027
8	0.159	0.0000	0.0617	0.0000
9	0.158	0.0049	0.0014	0.0262
10	0.157	0.0005	0.0121	0.0021
11	0.146	0.0449	0.0000	0.0023
12	0.144	0.0000	0.0039	0.0000

conditions consist of dead load (self-weight) of the structure and earthquake load defined by the response spectrum function for both directions. Before checking for stress distribution, displacement is observed. The maximum displacement for $G + EQ_x$ is $\Delta_x = 7.1$ mm and maximum displacement for $G + EQ_y$ is $\Delta_y = 8.0$ mm, as shown in Figure 12. The unit of measurement is mm.

In order to show more detailed information and analysis, displacement of the structure is observed through main elements of the roof system. Tables 5–7 provide more detailed displacement data for the top of the main dome, the top of small domes, and the top of the main arches of the structure. For each table, there is a figure to show the location of the observed values.

The seismic behavior of Bajrakli Mosque is explained through S_{22} vertical stresses which show the tensile and compressive stresses and S_{12} shear stresses. The results obtained through load combinations are shown visually and compared to ultimate limits defined previously. The following paragraphs show the results obtained from the analysis of the roofing system and walls separately for each of the loading conditions and shear stresses for tensile, compressive, and shear.

The S_{22} compressive and tensile stresses and S_{12} shear stresses for the roofing system are shown in Figures 16 and 17, respectively. The allowable tensile stress for stone masonry σ_m (tensile) = 0.135 MPa is exceeded at the roofing systems with a maximum value of 1.980 MPa for $G + EQ_x$ and 1.794 MPa for $G + EQ_y$. Same, the allowable compressive stress for stone masonry σ_m (compressive) = 0.9 MPa is exceeded with a value of 3.050 MPa for $G + EQ_x$ and 2.979 MPa for $G + EQ_y$. Figure 16 shows the S_{22} stress distribution, in which the darker colors are near or exceed the allowable limits. Note that the range of values (maximum and minimum) is decreased in order to emphasize more the areas of stress concentration. Values are shown in kN/m^2.

Recalling from previous section, the calculated allowable shear stress for stone masonry is $\tau_m = 0.53$ MPa. Shear stresses are represented with S_{12} values defined in the software. In the analysis for the loading conditions as $G + EQ_x$, and $G + EQ_y$, shear stresses exceed the allowable limit with a maximum value of 0.990 MPa and 1.090 MPa, respectively. Shear stress concentration is found to be higher in the pendentives of small domes and in the connections between them. The shear distribution of the structure for both loading combinations is shown at the following figure in which darker values show increase in stress. In order to emphasize more the areas of stress concentration, the range of values (maximum and minimum) is decreased. Values are shown in kN/m^2.

As the roof system results are received, the same procedure is applied for load-bearing wall analysis as well. The outputs obtained by the assessment of the walls using FE analysis show that the tensile stresses derived from S_{22} stresses exceed the allowable stresses for stone masonry σ_m (tensile) = 0.135 MPa with a maximum value 0.254 MPa for load combination in X direction and 0.255 in Y direction. Regarding the compressive stresses, the values derived from S_{22} do not exceed the allowable compressive stress σ_m (compressive) = 0.9 MPa. Maximum value is found to be 0.668 MPa for $G + EQ_x$, and 0.747 MPa for $G + EQ_y$. Regions with highest stress concentration are found to be near the openings such as windows and door. In the front wall, a high concentration of compressive stresses is shown from the analysis result. This is due to the openings and the connection of with the roof system of entrance hall. Figure 18 shows the S_{22} stress distribution for the walls, where the darker colors show increase in stress. Values are shown in kN/m^2.

Same as the roof system, walls are analyzed for shear values as well. The S_{12} values found from the analysis are compared to the limits defined. After the analysis, results show that for both load combinations $G + EQ_x$, and $G + EQ_y$, S_{12} values do not exceed the allowable shear stress $\tau_m = 0.53$ MPa. The maximum value for shear stresses in $G + EQ_x$ is 0.311 MPa, while the maximum value for shear stresses for $G + EQ_y$ is 0.291 MPa. Same as for tensile and compressive stresses, shear stress concentration is found to be higher near the openings of the load-bearing walls. By this result, it can be stated that openings affect the concentration of the shear stresses of the structure, thus impacting the behavior of the structural elements under loading conditions. In Figure 19, the shear stress S_{12} distribution through the walls is shown (darker colors represent higher values). The range of values is decreased in the figure to emphasize more the stress distribution around the structure.

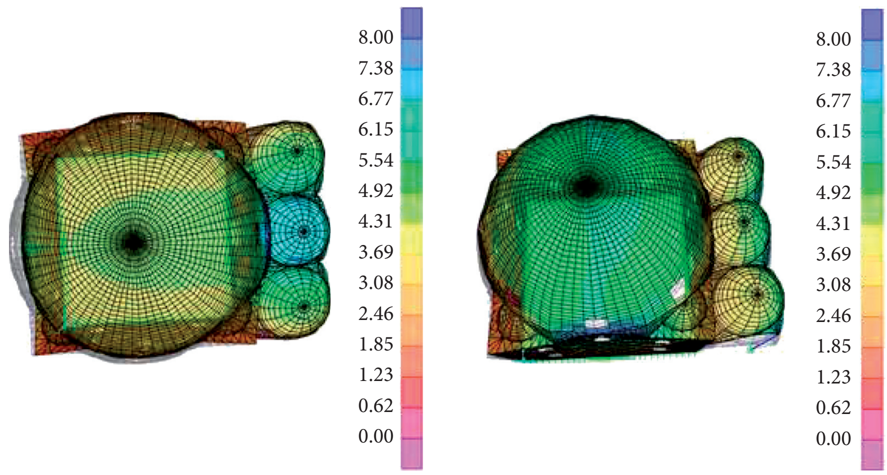


Figure 12: Maximum displacement for (a) $G + EQ_x$ and (b) $G + EQ_y$.

Table 5: Displacement values on top of the main dome (Figure 13).

Location	Displacement in $G + EQ_x$ (mm)			Displacement in $G + EQ_y$ (mm)		
	Horizontal X	Horizontal Y	Vertical Z	Horizontal X	Horizontal Y	Vertical Z
Top of the main dome	2.7	0.1	−4.6	0.8	2.3	−4.7

Table 6: Displacement values on the top of the small domes (Figure 14).

Location	Displacement in $G + EQ_x$ (mm)			Displacement in $G + EQ_y$ (mm)		
	Horizontal X	Horizontal Y	Vertical Z	Horizontal X	Horizontal Y	Vertical Z
Top of the small dome 1	6.0	0.8	−5.1	4.6	1.8	−4.9
Top of the small dome 2	7.1	0.1	−5.5	4.2	1.8	−5.2
Top of the small dome 3	6.1	0.8	−5.2	4.7	1.9	−4.9

Table 7: Displacement values on the top of the main arches (Figure 15).

Location	Displacement in $G + EQ_x$ (mm)			Displacement in $G + EQ_y$ (mm)		
	Horizontal X	Horizontal Y	Vertical Z	Horizontal X	Horizontal Y	Vertical Z
Top of the small dome 1	1.5	1.6	−3.5	0.6	2.5	−3.5
Top of the small dome 2	5.2	0.0	−2.7	2.1	1.5	−2.6
Top of the small dome 3	1.5	1.3	−3.9	0.6	2.3	−3.9
Top of the small dome 4	1.4	1.8	−2.2	0.5	3.7	−2.1
Top of the small dome 5	2.0	1.1	−3.1	1.4	1.7	−3.1
Top of the small dome 6	3.1	0.0	−2.1	0.5	1.1	−2.0
Top of the small dome 7	2.0	1.1	−3.0	1.3	1.8	−3.1
Top of the small dome 8	1.4	1.6	−2.2	0.4	3.6	−2.2

Table 8: Maximum values of S_{22} stress concentration of structure.

System	S_{22} surface	S_{22} stress	$G + EQ_x$ (MPa)	$G + EQ_y$ (MPa)
Roof	Top	Tensile	1.758	1.608
		Compressive	−2.980	−2.658
	Bottom	Tensile	1.980	1.794
		Compressive	−3.050	−2.979
Walls	Top	Tensile	0.220	0.233
		Compressive	−0.668	−0.747
	Bottom	Tensile	0.254	0.255
		Compressive	−0.583	−0.617

Table 9: Maximum values of S_{12} stress concentration of the structure.

System	S_{12} surface	$G + EQ_x$ (MPa)	$G + EQ_y$ (MPa)
Roof	Top	0.982	1.015
	Bottom	0.990	1.091
Walls	Top	0.311	0.291
	Bottom	0.307	0.274

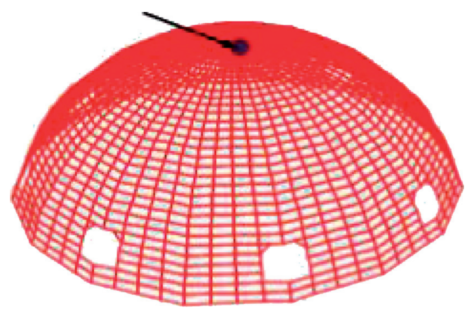


Figure 13: Location of the maximum displacements on the main dome.

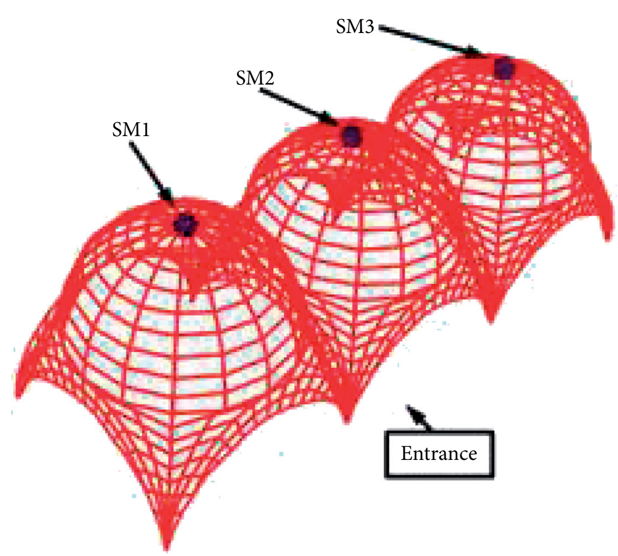


Figure 14: Location of the maximum displacements on the small domes.

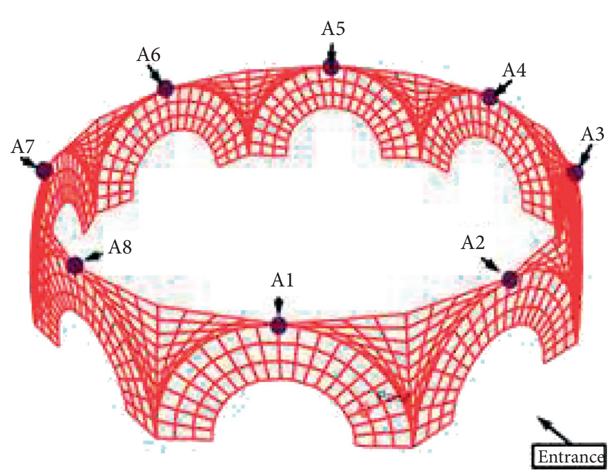


Figure 15: Location of the maximum displacements on the top of the main arches.

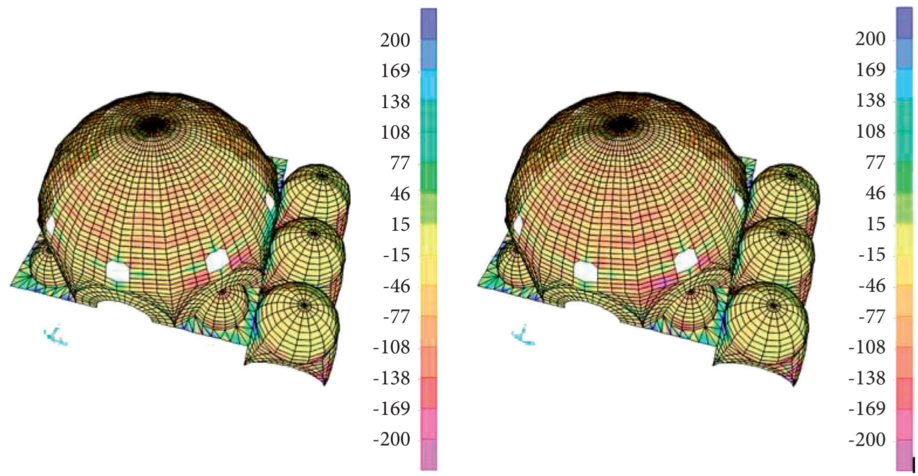


Figure 16: Tensile and compressive stress distributions at the roof system for (a) $G+EQ_x$ and (b) $G+EQ_y$.

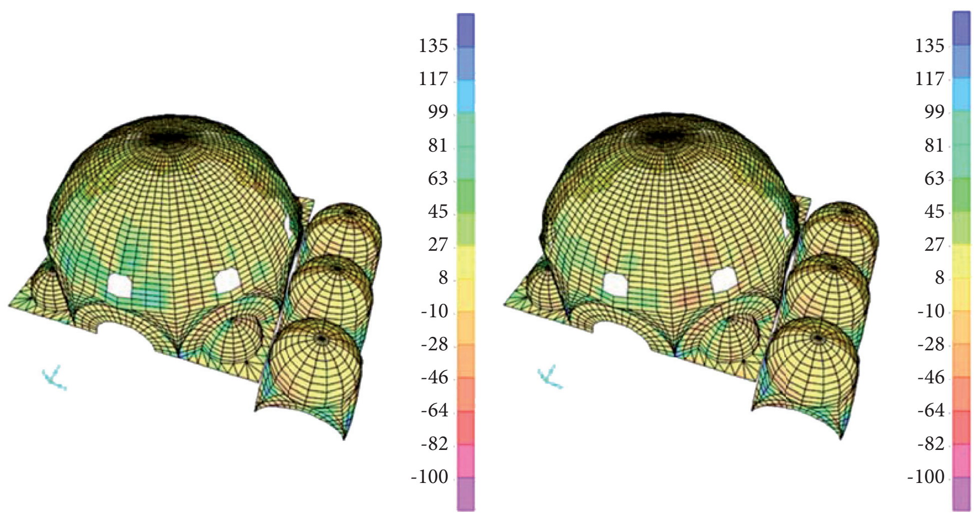


Figure 17: Shear stress distribution at the roof system for (a) $G+EQ_x$ and (b) $G+EQ_y$.

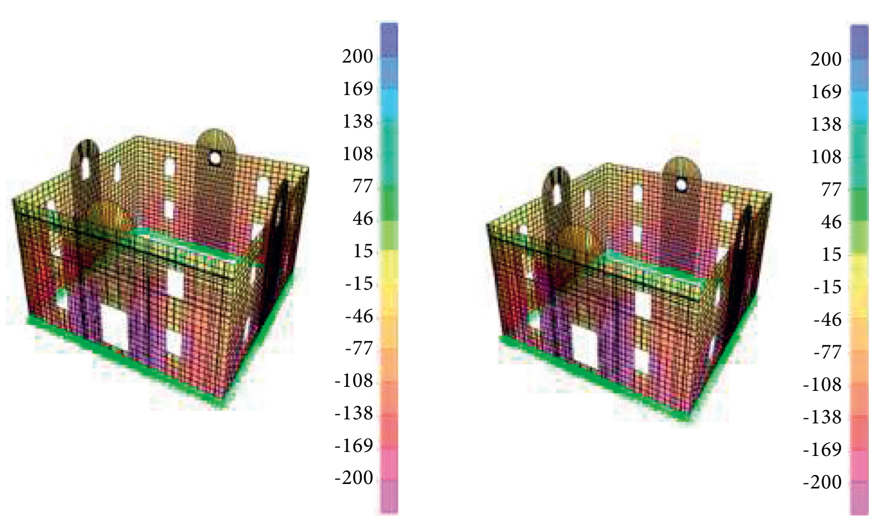


Figure 18: Tensile and compressive stress distribution at load-bearing walls for (a) $G+EQ_x$ and (b) $G+EQ_y$.

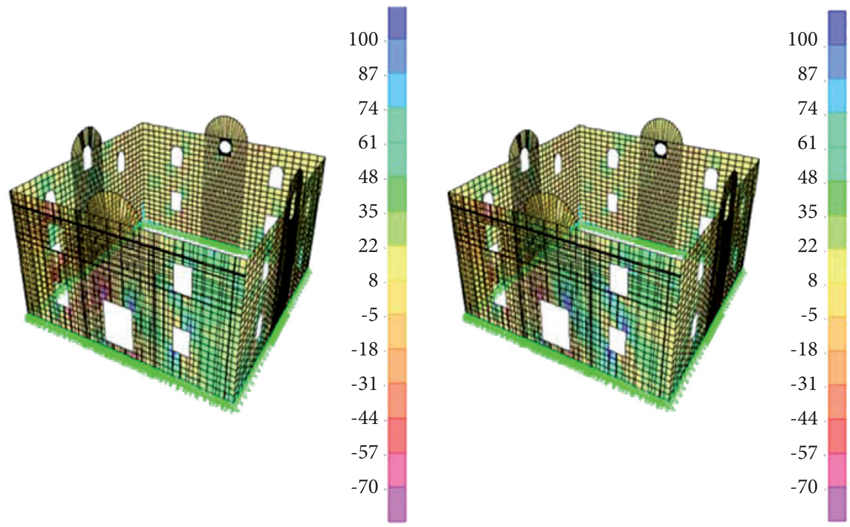


Figure 19: Shear stress distribution at load-bearing walls for (a) $G + EQ_x$ and (b) $G + EQ_y$.

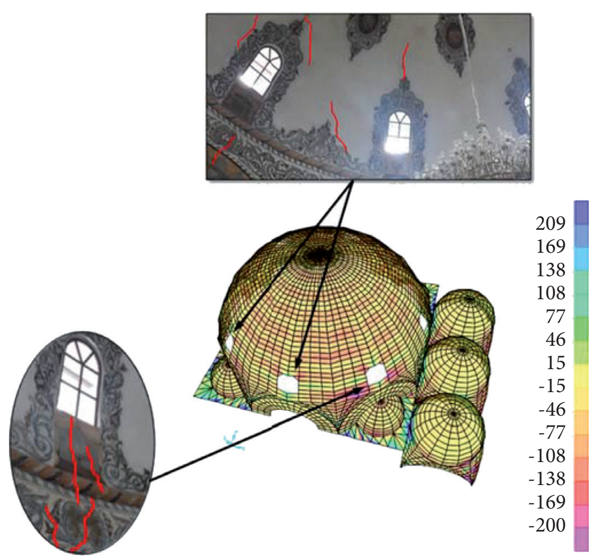


Figure 20: Comparison of visual inspections with finite element analysis for the roof system (S_{22}-$G + EQ_y$).

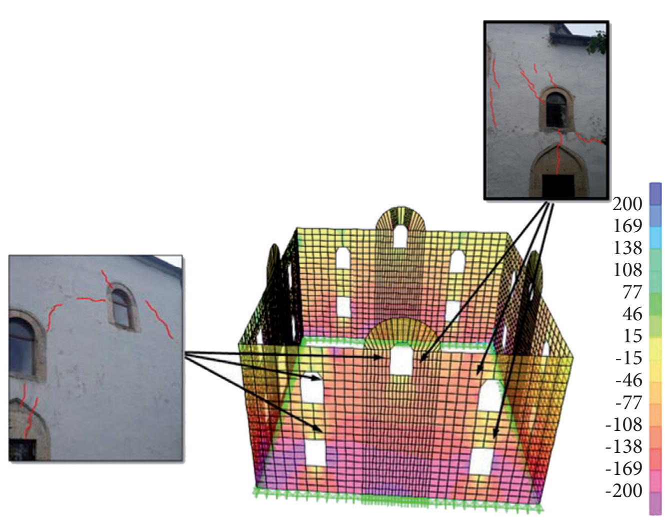


Figure 21: Comparison of visual inspections with finite element analysis for load-bearing walls (S_{22}-$G + EQ_x$).

From all the analysis done to the structure, the location with high stress concentration can be identified. For the roof system, the highest stress concentration is seen at small domes of the entrance halls, pendentives, and arches, and at the opening locations at the major dome. Similarly, for the wall system, the highest stress concentration is shown at the openings and at the connections with the arches of the small domes. Table 8 provides all the data regarding the compressive and tensile stresses for top and bottom surfaces for the roof system and load-bearing walls, under both load combinations as $G + \mathrm{EQ}_x$, and $G + \mathrm{EQ}_y$. Table 9 provides all the data for the S_{12} shear stresses for the roof and load-bearing walls for both loading conditions.

Thus, comparing to the cracks observed from visual inspections, it can be stated that the stress analysis match with the actual occurrence of the cracks through the structural elements, as shown in Figures 20 and 21.

8. Conclusions and Remarks

This study is conducted into two different methods. The first analysis of the structure is carried out by visual inspections, and the second method involves FEM analysis. Through visual inspections, cracks and other deficiencies of structure are observed and assessed. Bajrakli Mosque has gone through some architectural restoration; thus, not all deficiencies can be seen. However, cracks are found in structural elements such as domes, pendentives, arches, and walls, being emphasized in the openings of the structure.

In order to have a better identification of stress concentration, the FE model is prepared. Modeling of Bajrakli Mosque is done based on geometrical data stored by the Institute for Preservation of Historical Monuments. Some of the missing data are substituted and assumed from other studies with similar geometrical properties. For modeling of Bajrakli Mosque, macromodeling approach is used. Modeling phase is one of the most important phases of FEM analysis, since it determines the flow of loads for the applied loads. In other words, the results are dependent from the modeling of the structure. Different from other structures, mosques with geometrical shape as Bajrakli Mosque should be modeled starting from the roof system. Shell element mesh should be properly defined, so the flow through each element of the structure is correctly assured.

The FE model prepared by SAP2000 involved assumed material properties due to inability to conduct tests. The maximum displacement obtained from the analysis shows value of 7.1 mm for $G + \mathrm{EQ}_x$ and 8.0 mm for $G + \mathrm{EQ}_y$.

The results of the stresses in the roof system exceed the allowable limits defined in the study. The allowable tensile stress for stone masonry is exceeded at the roofing systems with a maximum value of 1.980 MPa for $G + \mathrm{EQ}_x$, while compressive stress limit is exceeded with a maximum value of 3.050 MPa for $G + \mathrm{EQ}_x$. Furthermore, shear stresses exceed the allowable limit with a maximum value of 1.090 MPa for $G + \mathrm{EQ}_y$ in the roof system. At the load-bearing walls, only allowable tensile stress is exceeded with a value of 0.255 MPa for load combinations in Y direction. The findings provided by finite element analysis results support the observations regarding the damage conditions through visual inspections.

Considering that this study was done based on material properties assumed through different research and studies similar to this structure and its construction period, the analysis of Bajrakli Mosque can be done using actual data which can be found by different laboratory tests and experiments. In addition, after all data and tests are gathered including foundation and soil properties, nonlinear analysis can be performed for similar structural monuments. Moreover, this study recommends that comparing various approaches for the analysis of historical monumental buildings is mandatory to cover unavoidable unknowns that may affect the response of materials and mechanics.

Data Availability

The research data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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